IOP Conf. Series: Earth and Environmental Science **134** (2018) 012026 doi:10.1088/1755-1315/134/1/012026

Catastrophe theory—one of the basic components in the analysis of the seismic response of rock mass to explosions

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Abstract. It is shown that the dynamic process of mining can be controlled using the catastrophe theory. The control parameters can be values of blasting energy and locations of explosions relative to an area under study or operation. The kinematic and dynamic parameters of the deformation waves, as well as the structural features of rock mass through which these waves pass act as internal parameters. The use of the analysis methods for short-term and medium-term forecast of rock mass condition with the control parameters only is insufficient in the presence of sharp heterogeneity. However, the joint use of qualitative recommendations of the catastrophe theory and spatial–temporal data of changes in the internal parameters of rock mass will allow accident prevention in the course of mining.

1. Introduction

The research into rock mass conditions is continued using the developed algorithm for processing data of detailed seismicity catalogue [1], providing supplemental information essential for predicting hazardous events in deep-level ore mining [2] with blasting. Furthermore, in spotlight is the process of initiation of resonant energy release, including the blasting impact and weak series energy responses in the form of tremors contributing to the resonant release. The message-bearing signs of initiation of a high-energy dynamic event are used: response time after an impact and volume of a rockburst source in the form of ranking of distance from the expected focus. The multimedia seismological information is of importance in prediction of hazardous events in ore mines.

The theoretical results on the causes of chaotization in nonlinear dissipative dynamic systems are compared with the data of processing detailed mine seismicity catalogue—seismic responses of rockburst-hazardous rock mass to blasting—by the method of phase diagrams. Theoretically, the common cause of the chaotization and stochastization in a dynamic system is the loss of stability, which can be recorded as exponential gapping between close-spaced phase trajectory, combined with the common boundedness and compaction. This result agrees with the phase diagrams plotted based on the mine seismicity catalogue data. [3, 4]. Currently, it is of significance that physics and mathematics start interacting, and needs of physics influence mathematical methods while mathematics has influence on physical knowledge. However, there are some physical problems for which the linear mathematics apparatus is either insufficient or inapplicable [5]. For all-around coverage of the diverse phenomena in acoustics and mechanics, the mathematical apparatus of linear differential equations is absolutely deficient and leaves aside the events which are the most specific and interesting. The fact is that the differential equations which describe these events are knowingly nonlinear. Accordingly, we talk of 'nonlinear' systems. The foundations of the mathematical apparatus relevant to individual problems and the whole cycle of nonlinear problems were laid by Poincaré and Lyapunov [6, 7].

In this study, the mathematical approaches of the catastrophe theory are applied to analyzing mine seismicity catalogue data on rock mass stability and instability under dynamic impact. One of the first researchers focusing on the loss of quasi-static equilibrium in rock mass using the catastrophe theory was Odintsev [8]. The important theoretical results were obtained in the sphere of estimation and prediction of dynamic conditions in rock mass under mechanical effect, and in assessment of post-dynamic condition, which is of specific significance in work planning in mine after rock burst. The application of these theoretical results needs stress monitoring at rock mass and mine interface and inside the rock mass, which is laborious. The way out is geophysical monitoring of wave fields containing information both on structure and stresses. One of such data bases is mine seismicity catalogue.

2. Review of the catastrophe theory methods for instability study in linear dynamic systems

The first studies on the catastrophe theory were published approximately in 1979 and told that this theory gives a universal tool to analyze all jump-type transitions, ruptures and qualitative changes [9]. According to Arnold [9], the sources of the catastrophe theory are Whitney' theory of smooth mapping and theory of bifurcation in dynamical systems by Poincaré and Andronov. Bifurcation means partition and describes qualitative restructuring of objects upon the change of parameters the objects depend on. Catastrophes are the jump-type changes as a spontaneous response of a system to smooth change in external conditions. The evolutionary process describing the system response to the applied effect is mathematically described by a vector field in a phase space [10]. The point in the phase space defines condition of the system, and the vector at this point denotes the rate of change in this condition. The points at which vectors turn to zero are named the balance positions. In course of time, vibration may arise in the system and equilibrium becomes unstable. On a phase plane, the stationary vibrations are represented by a closed curve called a limit cycle. The evolution of a dynamical system was studied in [9] with regard to time variations in internal and governing parameters. Generally, the application flowchart of the catastrophe theory starts with the assumption that the process under study is described using a certain number of governing and internal parameters. The equilibrium states of the process form a surface of some or other number of measurements in this space. The mapping of the surface of equilibriums onto plane of governing parameters has singularities called the singularities of general positions. The theory related with these singularities predicts geometry of 'catastrophes,' i.e. jumps between states under the change in the governing parameters. One of the most well-known singularities is Whitney's umbrella. After the loss of stability of equilibrium, two steady-state conditions can be observed. One is an oscillating periodic mode. This type of instability is called soft instability as the stabilizing oscillating mode, in case of small post-criticality, differs insignificantly from the equilibrium state. The second mode is connected with some features: before a steady-state mode loses stability, the domain of attraction of this mode becomes very small, and random disturbances throw the system from this domain before it vanishes. This type of loss in stability is called rough instability, and the system jumps from the stationary mode to another mode of motion. The latter can be another steady-state stationary mode, or steady-state vibrations, or event more complex motion. The steady-state modes of motion are attractors. The modes differing from equilibrium and strictly periodic oscillations are called strange attractors and connected with the problem of turbulence [10].

3. Analysis of the data of active mine seismicity monitoring using the catastrophe theory flowchart

The main objective of monitoring is detecting pre-critical condition of rock mass under mining with blasting. The pre-critical condition is defined as the response energy not higher than 10^4 J. The response energy in the range 10^5–10^7 J means the critical condition of rock mass. The response energy of 10^8–10^9 J and higher defines post-critical or catastrophic condition.

On Nov 25, 2007, in Tashtagol Mine cross-cut 29 at a depth of 714 m, rock burst with the energy of $8.14 \cdot 10^8$ J took place. Let us trace initiation of this event from the viewpoint of rock mass instability and connection with the effect and locations of blasting. The rock mass volume under study is represented in the coordinates *OX* (25–31 tunnel ≈ 240 m), *OY* (length of cross-cuts between fringe drift and air way is ≈ 240 m) and *OZ* (mine levels –140, –425 m, ≈ 590–875 m).

Below we analyze the morphology of time of rock mass responses using the data from the mine seismicity catalog since June 4, 2006 till November 25, 2007.

4 June–13 August 2006. The loss in equilibrium after four impacts takes places in the form of oscillations; the first blast is carried out inside the object under study but it has the least effect out of the four blasts, and the maximum energy response is within the pre-critical range.

20 August–17 September 2006. Classical mode of instability after two blasts. There is a response in the form of almost uniform time distribution of energy passing to oscillating process; Sep 3 blast energy exceeded 10^8 J, while Sep 17 blast energy exceeded 10^6 J thought the total energy of response reduces and is not higher than 10^3 J.

24 September–14 October 2006. The response has different morphology, the oscillating process changes to jump-type release follows by oscillation with the background level of soft instability. The next responses occur beyond the domain under study and behave as low-amplitude energy oscillations. On Oct 14, 2006, the seismic response morphology is the classical soft instability with the attenuation of oscillating energy release.

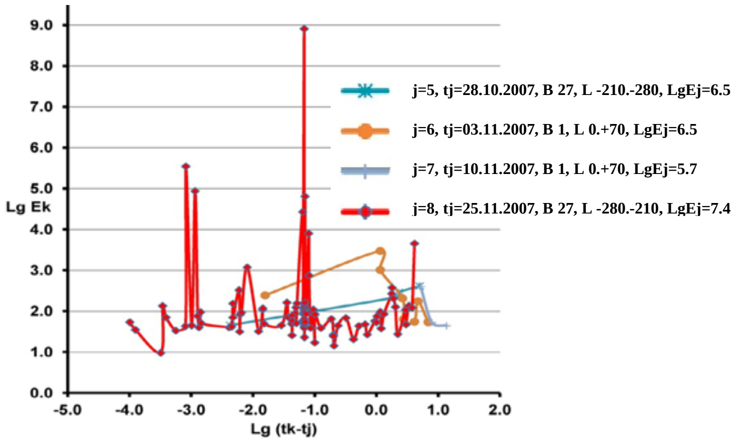

Figure 1. Morphology of energy *Ek* (J) of rock mass response to blasting in time from Oct 28 to Nov 25, 2007: *tk*—response time; *tj*—impact date and time (in days); *Ej*—disturbance energy (J), B—block; L—level

21 October–12 November 2006. Blasting is carried out inside the study domain but the morphology of soft instability is more pronounced after the blast on Nov 12. Immediately after the blast, oscillating energy release begins, then goes jump followed by oscillation and again jump with an amplitude more than 10^4 J. the Background energy oscillations exceed 10^2 J.

19 November–17 December 2006. Outside the study domain, a blast with the energy higher than 10^9 J is carried out, and the rock mass responds by two jump-type energy releases with the energy of the jumps on Nov 12, 2006. The next blast in the study domain on Nov 26, 2006 causes the soft instability response on the energy background of the second period of the analysis. The next blast on Dec 3, 2006 induces higher response with oscillations and jumps as in the third period of the analysis.

24 December 2006–21 January 2007. The blast inside the study domain (nearby the place of the future rock burst) with an energy of 10^8 J causes sharp though critical instability in the rock mass, two jumps with an energy under 10^5 J and three jumps with an energy of 10^4–10^3 J. The oscillations are more intensive. The other blasts in the distance from the study domain over the period from Dec 30, 2006 to Jan 14, 2007, sustain oscillations induced by the blast on Dec 24. The blast on Jan 21 with the energy lower than 10^5 J at a long distance from the study domain does not prevent jump-type energy release reflective of prolongation of stress relaxation period in the system.

25 March–8 April 2007. Blasting is conducted far from the study domain. The rock mass response is random and not oscillating, the energy is mostly not higher than 10^4 J, except for the response to the blast on Apr 1, 2007 when the energy jumps over 10^4 J.

6–20 May 2007. Basting in the north of the rock mass, than, on May 13 simultaneous blasting in the north and inside the study domain (in the cross-cut where the catastrophic rock burst occurred later on), rock mass response is a jump with an energy of 10^5 J.

13 June–15 September 2007. The study domain mostly responds to the outside blasts, the response to the internal blasting is jumps of energy to 10^5 J without preliminary oscillating.

30 September–21 October 2007. Blasting only in the study domain nearby the rock burst cross-cut. The energy distribution morphology shows the low-energy dependence on time and very short period of oscillations.

It is seen in the figure above that blasting between October 28 and November 10 induces no oscillating energy response in rock mass. The blasting on Nov 25, 2007, in block 27 causes a complex dynamic catastrophe process. There are three foreshocks, one aftershock, four pre-critical and one post-critical jump of energy. All these events are accompanied by low-energy oscillations.

4. Conclusion

Mining is a dynamic process which is controllable using the recommendations of the catastrophe theory. The governing parameters of the process can be blast energy and blast locations relative to the rock mass domain subjected to the analysis. The internal parameters are the kinematic and dynamic characteristics of deformation waves [11, 12] and the rock mass structure where the waves propagate [13]. It is insufficient to make short-term and long-term prediction of sharply nonuniform rock mass conditions sing only the governing parameters. On the other hand, the joint application of the qualitative recommendations of the catastrophe theory and the data on space and time of change in internal parameters will make it possible to prevent catastrophes in mines.

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